Randomness is sufficient! Approximate recovery from CS samples

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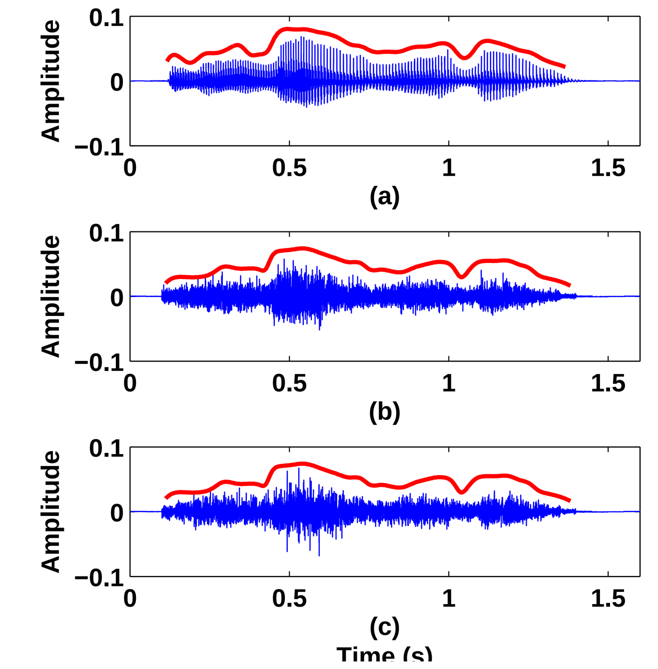
Introduction 1

- In CS, for efficient recovery sensing matrix must preserve the relative distances among the underlying sparse vectors.
- Hence, for a signal sampled in time domain, CS samples will also preserve its envelope.
- We propose to extract prototype signals from compressive samples to build the dictionary to obtain sparse representation for approximate recovery of the actual signal.
- Prototype signals are orthogonal intrinsic mode functions (IMFs) extracted using empirical mode decomposition (EMD).



(a) Speech, (b) and (c) CS Speech with compression ratio .6 and .4

Proposed Approach 2

Basic model: In CS framework, a signal is measured using non-adaptive linear measurements as

$$\mathbf{y} = \mathbf{\Phi x} = \mathbf{\Phi \Psi}\alpha = \mathbf{D}\alpha, \quad \text{or in matrix form} \quad \mathbf{Y} = \mathbf{\Phi X}, \quad \text{assuming} \quad \mathbf{x} = \mathbf{\Psi}\alpha$$

Signal is recovered as

$$\hat{\alpha} = \operatorname{argmin} \|\alpha\|_1 \quad \text{subject to} \quad \|\mathbf{y} - \mathbf{D}\alpha\|_2^2 < \epsilon$$

$$\|\mathbf{D}(\alpha_1 - \alpha_2)\|_2^2 \approx \|\alpha_1 - \alpha_2\|_2^2 \;\forall\; \alpha_1, \alpha_2$$

$$\hat{\mathbf{\Sigma}} = \mathbf{YY^T} = \mathbf{\Phi XX^T\Phi^T} = \mathbf{\Phi\Sigma\Phi^T} \text{and}$$

$$\mathbf{E}\left[\|\mathbf{\Phi x}\|_2^2\right] = \|\mathbf{x}\|_2^2$$

Assumption: No training data available; Resource-constrained environment.

Proposed Method: Using EMD, a given compressed signal $\mathbf{y}$ is expressed as a sum of J orthogonal modes $\mathbf{m}_q$ and a residual $\mathbf{r}$

$$\mathbf{y} = \sum_{q=1}^{J} \mathbf{m}_q + \mathbf{r}$$

Proposed Solution

- Cosine interpolate $\mathbf{Y}$ • Compute J IMFs using EMD of each frame.
- Cluster all IMFs across J^{th} level. • Build dictionary using cluster centers.

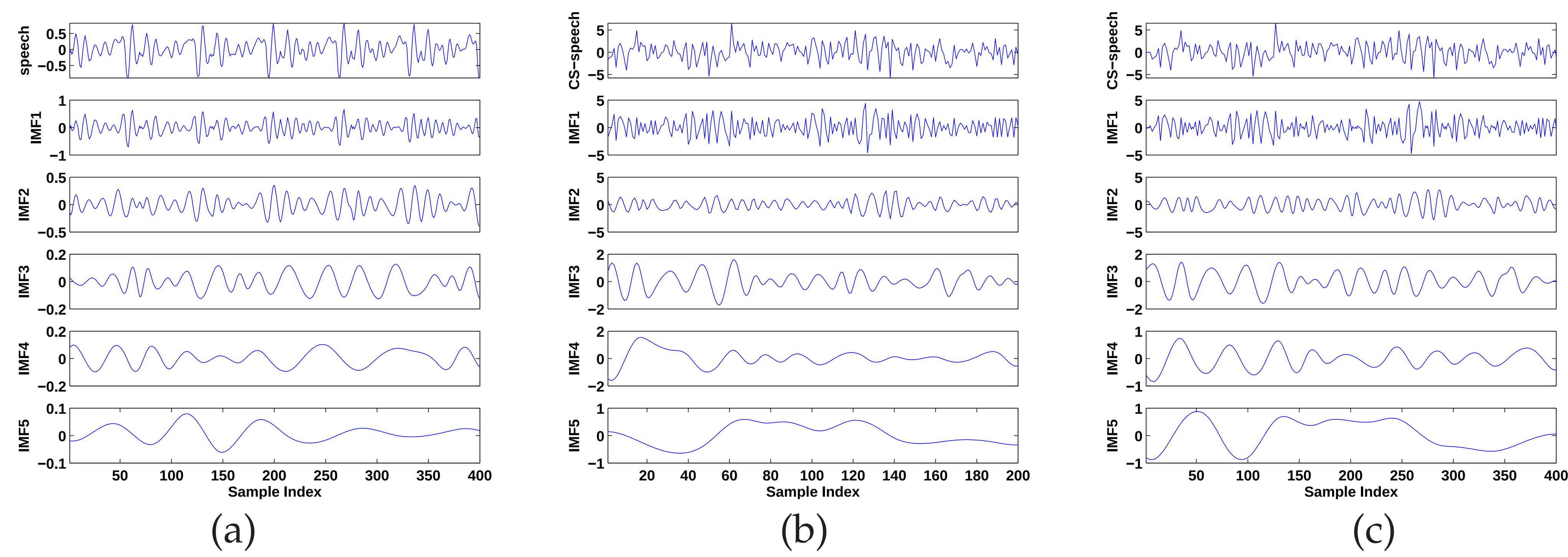
Guess what we don't need $\mathbf{\Phi}$ for DL

Advantages: • Fast Reconstruction • Near Real-time Decoding • Inference based on extracted IMFs.

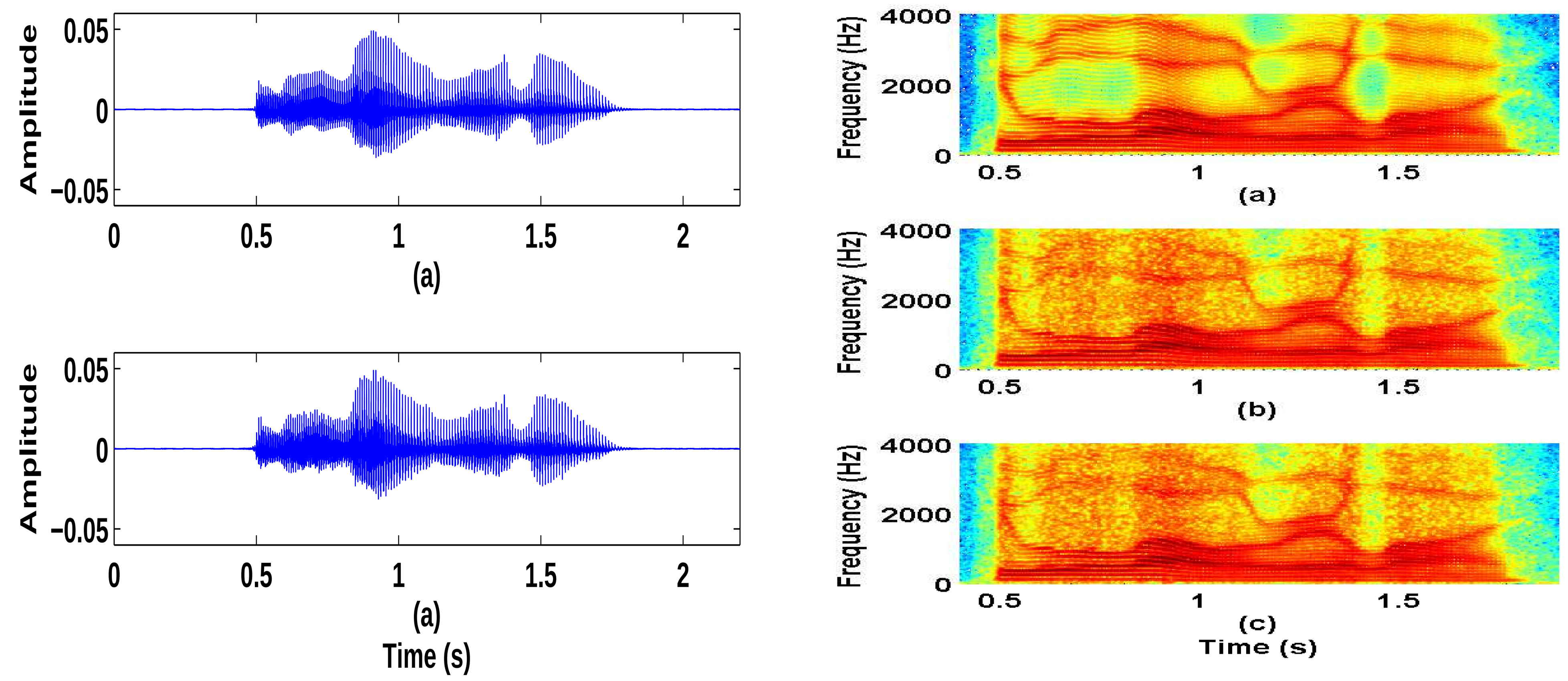
Any Alternatives: Yes!
Apply sub-band filtering instead of EMD to save computation.

Abbreviations: $\mathbf{\Psi}$ - Dictionary | $\mathbf{\Phi}$ - Measurement Matrix | $\mathbf{D}$ - Reconstruction Matrix | $\mathbf{x}$ - Original Signal | $\mathbf{y}$ - Compressed Signal | α - Sparse Vector | $\mathbf{m}$ - IMF |

Results 3



EMD decomposition of a voiced frame of (a) original speech (b) compressive speech and (c) interpolated compressive speech signal



Reconstruction Results in Time Domain and Frequency Domain

Comparative Analysis of Different Methods for Signal Recovery averaged for 20 utterances over 10 trials.

Method	CS Matrix	DL Iterations	PESQ	Runtime
CS-EMD	SRM	N.A	2.92	0.83 min
	Gaussian		2.90	
	Bernoulli		2.84	
CS+DCT	Gaussian	N.A	2.30	0.3 min
Blind CS	Gaussian	20	2.97	5 min
IHT	Gaussian	20	3.10	3 min

Blind CS-Gleichman et al.; IHT-Srikanth et al.; SRM-Thong et al.

References 4

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