Simultaneous Reconstruction and Separation in a Spectral CT Framework

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Introduction

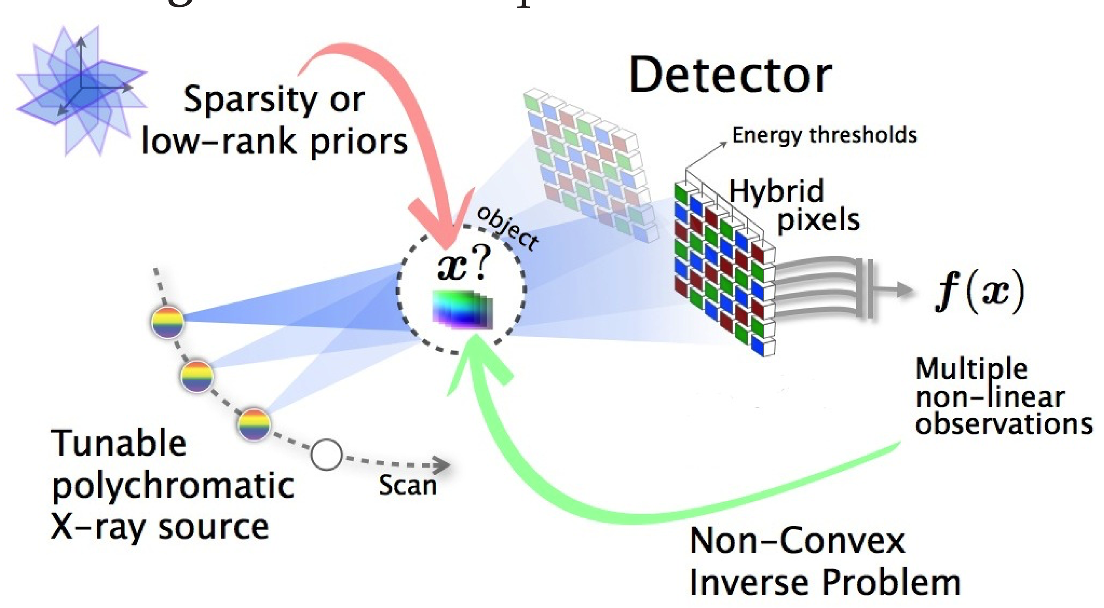
The advent of new X-ray detection technology by hybrid pixel cameras working in a photon-counting mode paves the way to the development of **spectral CT**.

This allows to simultaneously separate and reconstruct the physical components of an object and finds applications in many areas of imaging.

Our framework allows to iteratively reconstruct an image from a spectral CT data by setting a **polychromatic model** that encompasses constraints (positivity, sparsity) and solving an **ill-posed inverse and non-convex problem**.

Preliminary results are obtained on simulated images containing four elements (water, Iodine, Yttrium and Silver).

Figure 1: From acquisition to resolution.



Forward model

Acquisition

A Computerized-Tomography (CT) scan is obtained by shining a X-Ray light modulated by metallic filters through a rotating object. **In the spectral setting, one explicitly exploits the spectral polychromaticity of the source attenuated through different metallic filters.**

The Beer-Lambert law governs the measurements:

$$y^m = \int_{\mathbb{R}+} f^m(E) e^{-\int_{\mathcal{L}^p} \mu(l,E)dl} dE \qquad (1)$$

where $f^m(E) = I_0(E) Fi^q(E) D^r(E)$ denotes the total spectral inputs:

- $\mu(l,E)$**:** absorption coefficients of the object for l on the line of sight $\mathcal{L}^p$ (to be found),
- I_0: X-ray source's intensity energy spectrum,
- Fi: filter's attenuation energy spectrum,
- D: detector's efficiency energy spectrum.

Absorption maps model

The absorption maps are naturally the sums of the contributions of each of their components; moreover the **spectral signature is physically independent of the spatial location** of a component:

$$\mu(l,E) = \sum_{k=1}^{K} a^k(l)\sigma^k(E), \qquad (2)$$

with $a^k(l)$ the concentration of component k at point l, and $\sigma^k(E)$ its interaction cross section. Eq. (1) now reads:

$$y^m = \int_{\mathbb{R}+} f^m(E) e^{-\sum_{k=1}^{K} \sigma^k(E) \int_{\mathcal{L}^m} a^k(l)dl} dE \qquad (3)$$

Discretized forward model

The energy E is discretized in N bins, and the 3D-volume where the object lives in D voxels. The forward discretized model reads:

$$Y = (F \odot e^{-SA\Sigma})\mathbb{1}_N, \qquad (4)$$

- $Y \in \mathbb{R}^M$: discretized measured data;
- $F \in \mathbb{R}^{M\times N}$: dictionary of energy modulating filters;
- $S \in \mathbb{R}^{M\times D}$: X-ray Transform operator;
- $A \in \mathbb{R}^{D\times K}$: concentration matrix: $A[d,k] = a_k(v_d)$;
- $\Sigma = (\sigma_1(.), \ldots, \sigma_K(.))^T \in \mathbb{R}^{K\times N}$: dictionary of the interaction cross sections of the K components: $\Sigma[k,n] = \sigma_k(E_n)$.

($\mathbb{1}_N = (1, ..., 1)^T$ and $\odot$ is the Hadamard product.)

Method

The measurements Y are noisy realizations of the perfect measurement described by Eq. (4). The inverse problem of recovering A, the matrix containing the concentration coefficient maps of the K components of the object, is solved by minimizing:

$$J(A) = D(Y, (F \odot e^{-SA\Sigma})\mathbb{1}_N) + R(A) \qquad (5)$$

with

- $D(Y,Z)$ a discrepancy measure (negative log-likelihood for Gaussian or Poisson noise),
- $R(A)$ a regularization term that models our a priori on A.

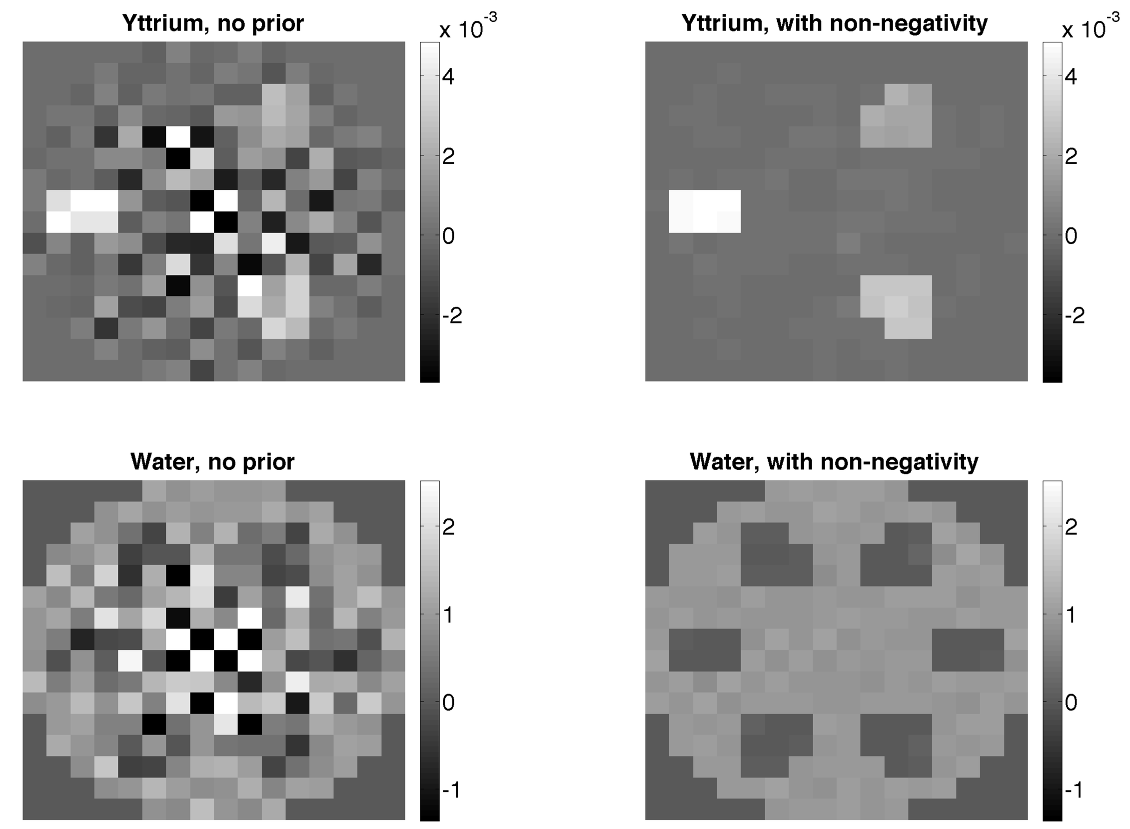
With $R(A)$ the non-negativity constraint, Eq. (5) is minimized with a classical trust-region algorithm

Trust-Region Algorithm

Require: A_0, Δ_0
  for $k = 1, 2, 3..$ **do**
    $m_k(p) = J(A_k) + g_k^T p + \frac{1}{2} p^T B_k p$ ▷ Quadratic model.
    $p_k \leftarrow \underset{\|p\|<\Delta_k}{ArgMin} \; m_k(p)$ ▷ Step calculation.
    $R_K \leftarrow \frac{f(X_k) - f(X_k + p_k)}{m_k(0) - m_k(p_k)}$ ▷ Ratio actual/predicted reduction.
    if $R_k acceptable$ **then**
      $A_{k+1} \leftarrow A_k + p_k$
      $Update\Delta_{k+1}$
    else
      $Reduce\Delta_{k+1}$
    end if
  end for
  return A_k

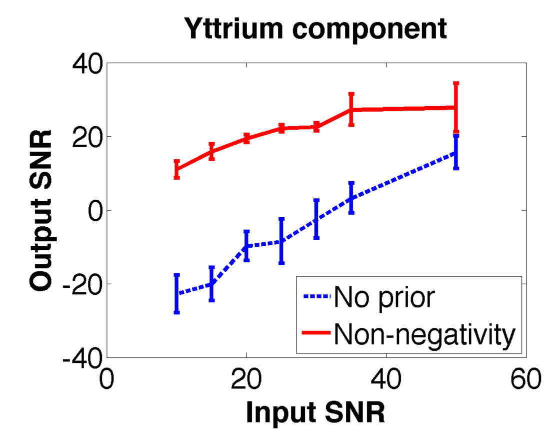
Results

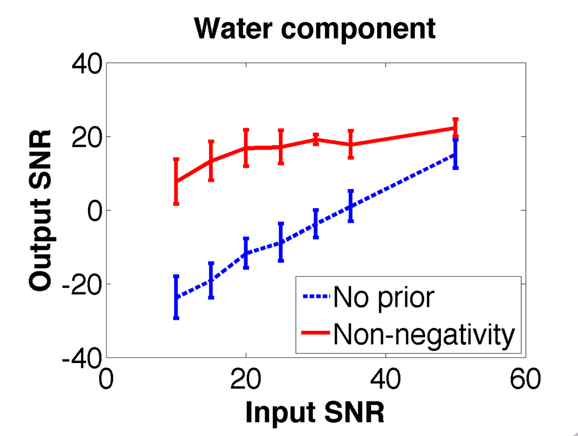
We have generated a **contrast phantom** made of one large cylinder filled with water and six smaller tubes filled with contrast agents. Three tubes contain Yttrium at different concentrations, two contain Silver and one contains Iodine ($K = 4$).



We have simulated a set of tomographic scans smeared by Gaussian noise with **5 different metallic filters** Fi, which are ideal pass-band filters around the discontinuities specifying the contrast agents. The discretized sizes are $M = 1440$, $D = 256$, and $N = 43$.

We have then minimized Eq. (5) with the non-negativity constraint using a trust-region algorithm and report the results obtained for 10 realizations of noise. The figures show that the low-rank model allows to reconstruct simple maps from non-linear polychromatic measurements. The output SNR evolves linearly with the input SNR.





Conclusion

We have established a new flexible model for spectral CT reconstruction. Results obtained with a classical Trust-Region approach on simulated data pave the way to future developments including sparsity constraints.