Blind Deconvolution of PET Images using Anatomical Priors

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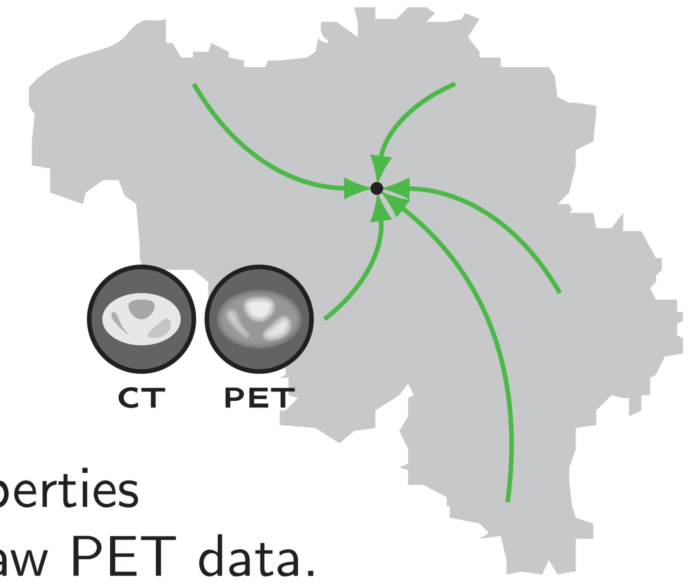
Goal. Improvement of the quality of positron emission tomography (PET) images for a better delineation of tumor volumes.

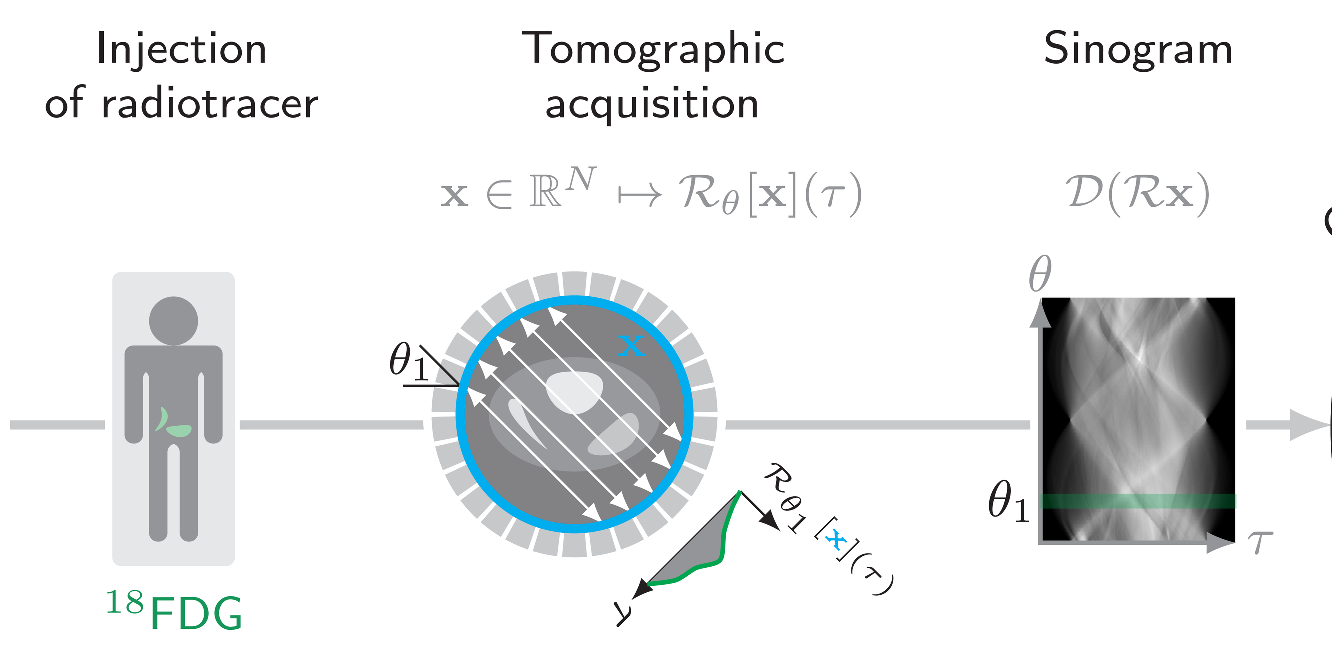
Context. Access to reconstructed images from different clinical centres and raw anatomical images from combined PET/CT scanners.



Challenge. No access to scanners properties (*e.g.*, the point spread function) and raw PET data.

Acquisition process

External and internal factors degrade the image resolution.



Forward model

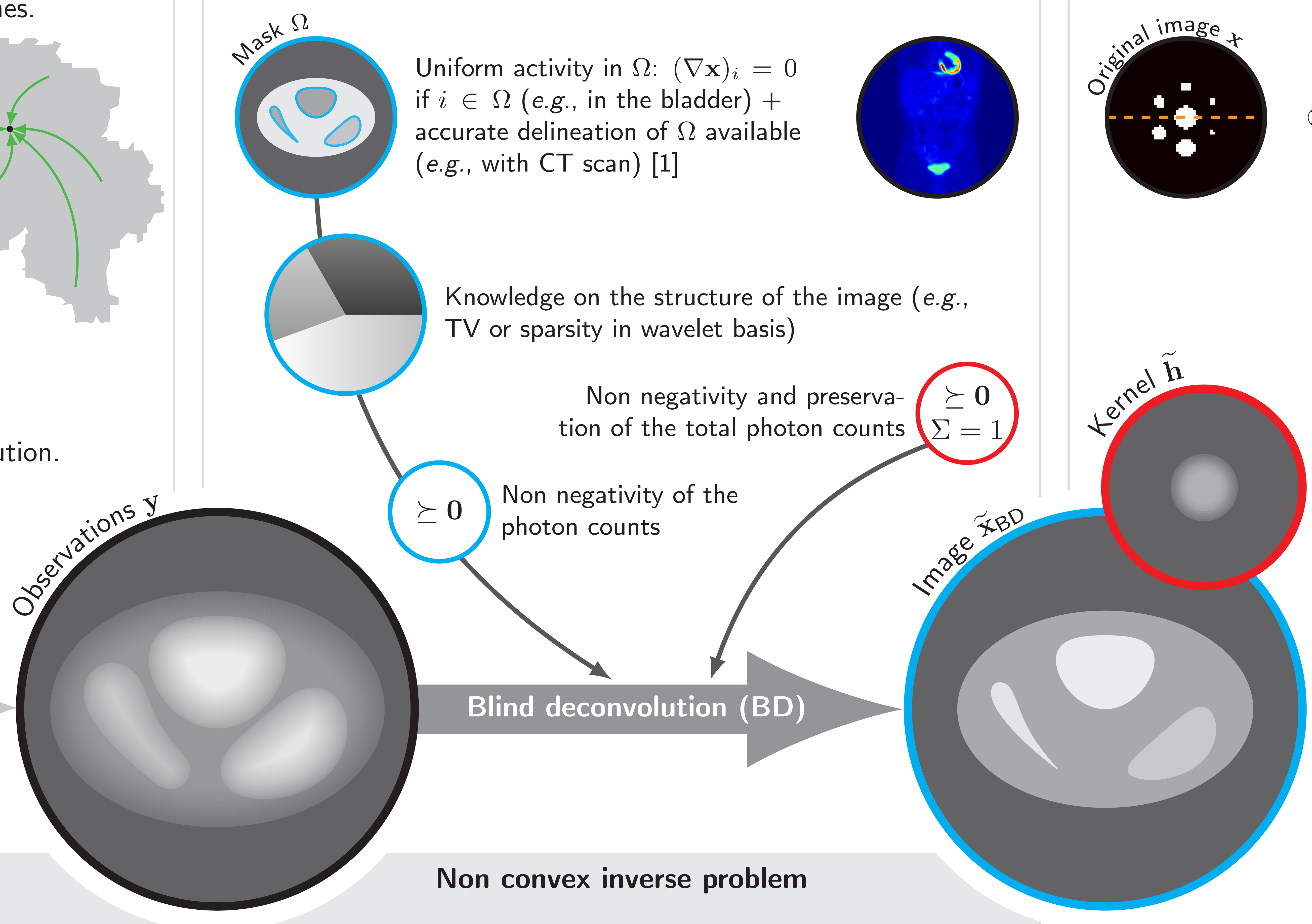
$$\mathbf{y} = \mathbf{h} \otimes \mathbf{x} + \boldsymbol{\eta}$$

Assumptions

- $\mathbf{h} \in \mathbb{R}^N$: unknown convolution operator (linear and uniform)
- $\boldsymbol{\eta} \in \mathbb{R}^N$: additive, white and Gaussian noise

How to simultaneously estimate $\mathbf{h}$ and restore image $\mathbf{x}$ from $\mathbf{y}$?

Prior knowledge on the image and the PSF



Uniform activity in Ω: $(\nabla\mathbf{x})_i = 0$ if $i \in \Omega$ (*e.g.*, in the bladder) + accurate delineation of Ω available (*e.g.*, with CT scan) [1]

Knowledge on the structure of the image (*e.g.*, TV or sparsity in wavelet basis)

Non negativity and preservation of the total photon counts

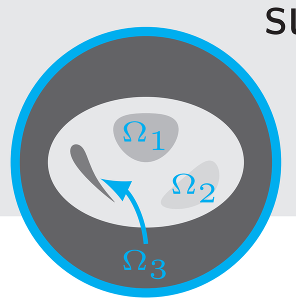
Non negativity of the photon counts

Non convex inverse problem

$$\underset{\mathbf{x},\mathbf{h}}{\text{minimize}} \quad \tfrac{\rho}{2}\|\mathbf{h}\otimes\mathbf{x}-\mathbf{y}\|_2^2 + \mathrm{TV}(\mathbf{x})$$

$$\text{subject to} \quad (\nabla\mathbf{x})_i = 0 \quad \text{if } i \in \Omega_1, \Omega_2, \ldots,$$

$$\mathbf{x} \succeq \mathbf{0}, \quad \mathbf{h} \succeq \mathbf{0}, \quad \textstyle\sum_{i=1}^N |h_i| = 1.$$



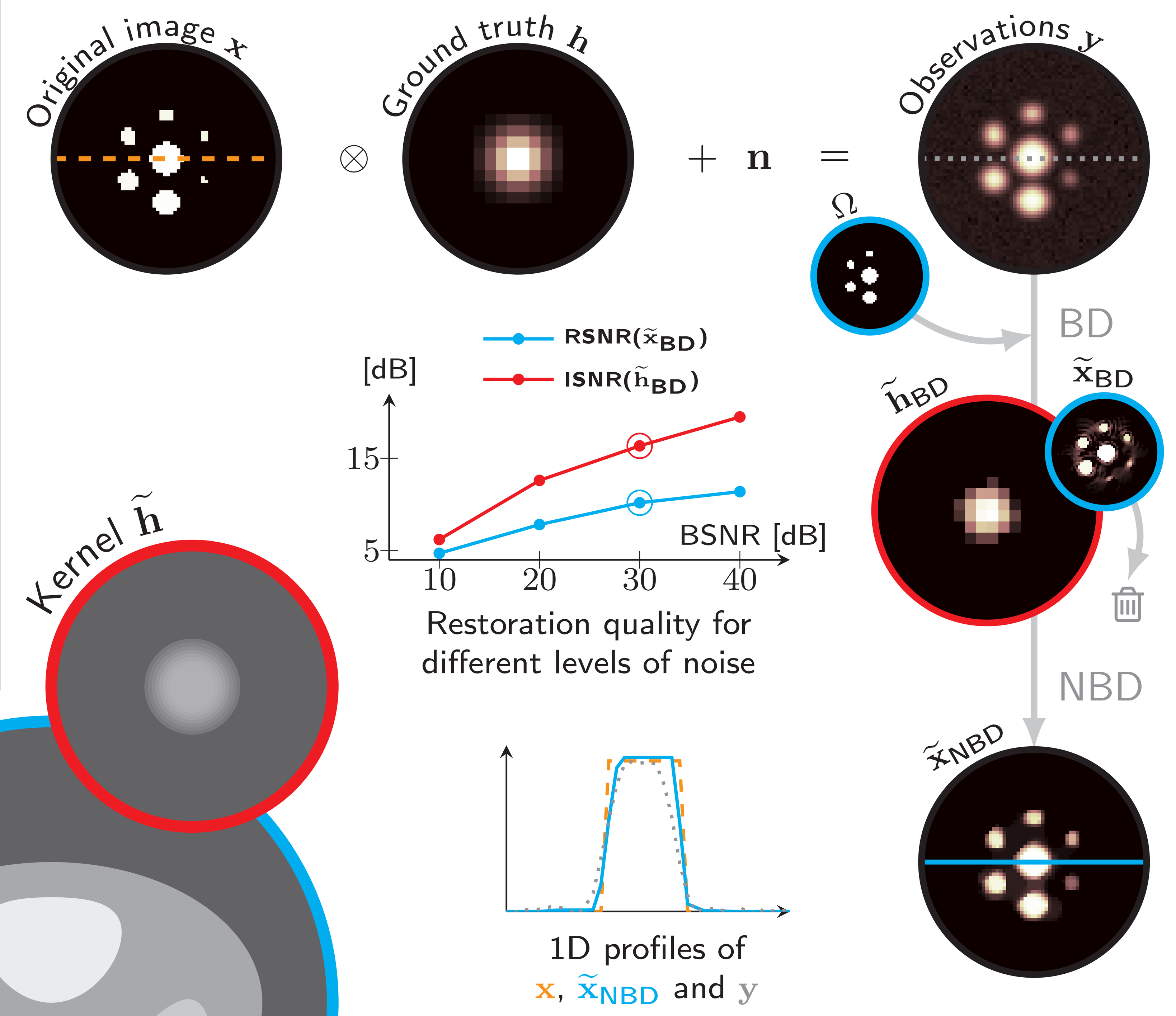
Solved through an alternated minimization [2,3]:

$$(\mathbf{x}^{(k)}, \mathbf{h}^{(k)}) \to (\mathbf{x}^{(k+1)}, \mathbf{h}^{(k)}) \to (\mathbf{x}^{(k+1)}, \mathbf{h}^{(k+1)})$$

$$\begin{cases} \mathbf{x}^{(k+1)} = \operatorname{argmin}_{\mathbf{x}} \ L(\mathbf{x}, \mathbf{h}^{(k)}) + \frac{\lambda_x^{(k)}}{2}\|\mathbf{x}-\mathbf{x}^{(k)}\|_2^2 \\ \mathbf{h}^{(k+1)} = \operatorname{argmin}_{\mathbf{h}} \ L(\mathbf{x}^{(k+1)}, \mathbf{h}) + \frac{\lambda_h^{(k)}}{2}\|\mathbf{h}-\mathbf{h}^{(k)}\|_2^2 \end{cases}$$

- $L(\mathbf{x}, \mathbf{h})$: objective function including cost function and constraints
- λ_x, λ_h: cost-to-move parameters

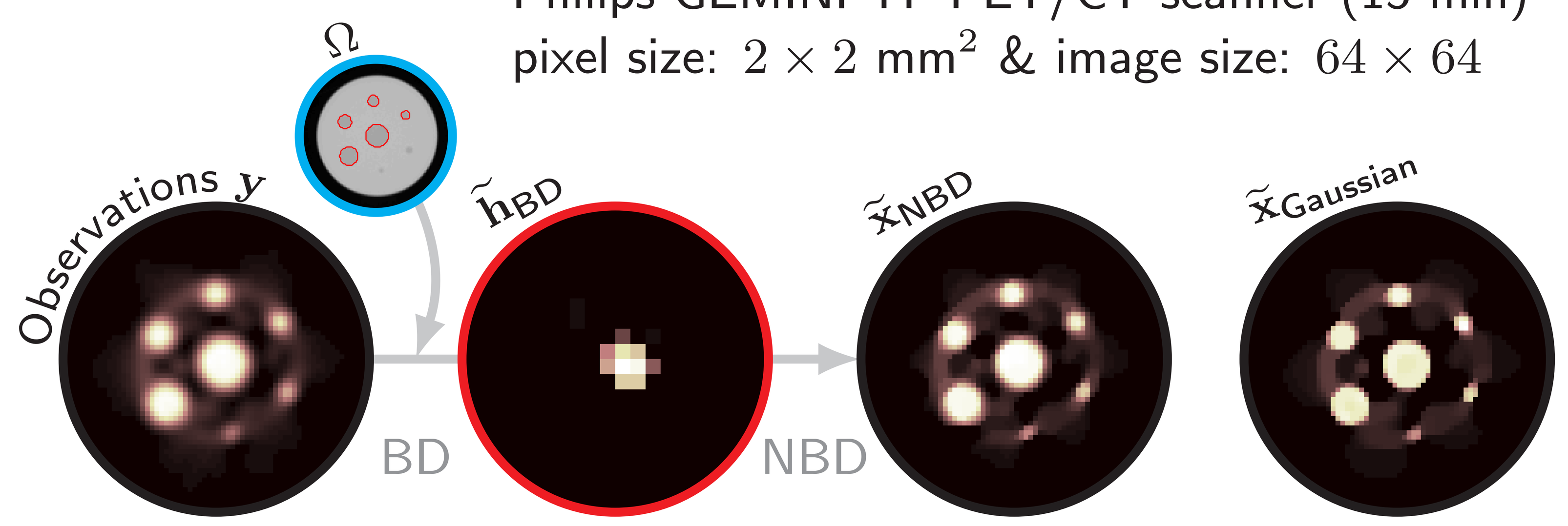
Validation and results on synthetic data



1. BD with priors on Ω: $\tilde{\mathbf{h}}_{\text{BD}}$ and $\tilde{\mathbf{x}}_{\text{BD}}$
2. NBD of new $\mathbf{y}$ without prior on Ω: validation if $\tilde{\mathbf{h}}_{\text{BD}}$ leads to regions with vanishing gradient

Results on real data

Phantom properties: cylindrical holes filled with ^{18}FDG
Philips GEMINI-TF PET/CT scanner (15 min)
pixel size: 2×2 mm^2 & image size: 64×64



Main references

[1] A. González, V. Delouille and L. Jacques, "Non- parametric PSF estimation from celestial transit solar images using blind deconvolution," *J. SpaceWeather Space Clim.*, 6(A1), 2016.
[2] A. Chambolle and T. Pock, "A First-Order Primal-Dual Algorithm for Convex Problems with Applications to Imaging," *Journal of Mathematical Imaging and Vision*, 40(1):120–145, 2010.
[3] H. Attouch, J. Bolte, P. Redont and A. Soubeyran, "Proximal Alternating Minimization and Projection Methods for Nonconvex Problems: An Approach Based on the Kurdyka-Łojasiewicz Inequality," *Math. Oper. Res.*, 35(2):438–457, 2010.

Future work

- Poisson noise and mixture Poisson/Gaussian noise
- 3D volume instead of 2D reconstruction per slice
- Extra constraints on the kernel (*e.g.*, sparsity in wavelet basis)
- Work with patient data